Conservation Laws and Generalized Isometric Embeddings

Nabil Kahouadji, Ph.D.

(McGill University)

Workshop on Moving Frames in Geometry Centre de Recherches Mathématiques Montréal, Canada

June 14, 2011

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- I. Conservation Laws
- II. Generalized Isometric Embedding Problem

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- I. Conservation Laws
- II. Generalized Isometric Embedding Problem
- III. Two Fundamental Motivations

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- IV. Results

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- V. Solving Strategy

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- II. Generalized Isometric Embedding Problem
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- V. Solving Strategy
- VI. Perspectives

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 $\mathcal{F} \longrightarrow \Gamma(\mathrm{T}\mathcal{M}^m)$

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$\mathcal{F} \longrightarrow \Gamma(\mathrm{T}\mathcal{M}^m)$

 $\{\mathsf{Solutions}\ \mathsf{PDE}\} \mapsto \{\mathsf{divergence-free}\ \mathsf{tangent}\ \mathsf{vector}\ \mathsf{fields}\}$

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$\mathcal{F} \longrightarrow \Gamma(\mathrm{T}\mathcal{M}^m)$

 $\{\mathsf{Solutions}\ \mathsf{PDE}\} \mapsto \{\mathsf{divergence-free}\ \mathsf{tangent}\ \mathsf{vector}\ \mathsf{fields}\}$

2 (\mathcal{M}^m, g) Riemannian manifold.

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 $\{\mathsf{Solutions}\ \mathsf{PDE}\}\mapsto\{\mathsf{divergence-free}\ \mathsf{tangent}\ \mathsf{vector}\ \mathsf{fields}\}$

2 (\mathcal{M}^m, g) Riemannian manifold. Then $\operatorname{div} X.\operatorname{Vol}_g = \operatorname{d}(X \lrcorner \operatorname{Vol}_g)$

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 $\{\mathsf{Solutions}\ \mathsf{PDE}\}\mapsto\{\mathsf{divergence-free}\ \mathsf{tangent}\ \mathsf{vector}\ \mathsf{fields}\}$

2 (\mathcal{M}^m, g) Riemannian manifold. Then $\operatorname{div} X.\operatorname{Vol}_g = \operatorname{d}(X \lrcorner \operatorname{Vol}_g)$

$$\mathcal{F} \longrightarrow \Gamma(\wedge^{m-1}\mathrm{T}^*\mathcal{M}^m)$$

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 $\{\mathsf{Solutions}\ \mathsf{PDE}\}\mapsto\{\mathsf{divergence-free}\ \mathsf{tangent}\ \mathsf{vector}\ \mathsf{fields}\}$

2 (\mathcal{M}^m, g) Riemannian manifold. Then $\operatorname{div} X.\operatorname{Vol}_g = \operatorname{d}(X \lrcorner \operatorname{Vol}_g)$

 $\mathcal{F} \longrightarrow \Gamma(\wedge^{m-1} \mathrm{T}^* \mathcal{M}^m)$ {Solutions PDE} \mapsto {Closed differential (m-1)-forms}

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 $\{\mathsf{Solutions}\ \mathsf{PDE}\}\mapsto\{\mathsf{divergence-free}\ \mathsf{tangent}\ \mathsf{vector}\ \mathsf{fields}\}$

2 (\mathcal{M}^m, g) Riemannian manifold. Then $\operatorname{div} X.\operatorname{Vol}_g = \operatorname{d}(X \lrcorner \operatorname{Vol}_g)$

$$\mathcal{F} \longrightarrow \Gamma(\wedge^{m-1} \mathrm{T}^* \mathcal{M}^m)$$

{Solutions PDE} \mapsto {Closed differential (m-1)-forms}

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$$\mathcal{F} \longrightarrow \Gamma(\wedge^{p} \mathrm{T}^{*} \mathcal{M}^{m})$$

{Solutions PDE} \mapsto {Closed differential p-forms}

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Generalized Isometric Embedding Problem

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Generalized Isometric Embedding Problem

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 $g, \nabla \quad \mathbb{V}^n$

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Genralized Isometric Embedding Problem (F. Hélein)



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Genralized Isometric Embedding Problem (F. Hélein)

$$g, \nabla \quad \mathbb{V}^{n}$$
$$d_{\nabla} \phi = 0$$
$$\phi \in \Gamma(\mathbb{V}^{n} \otimes \wedge^{p} T^{*} \mathcal{M}^{m})$$

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Genralized Isometric Embedding Problem (F. Hélein)

$$g, \nabla \quad \mathbb{V}^{n} \underbrace{ \Psi? }_{\mathsf{d}_{\nabla}} \phi = 0$$

$$\phi \in \Gamma(\mathbb{V}^{n} \otimes \wedge^{p} \mathrm{T}^{*} \mathcal{M}^{m})$$

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Genralized Isometric Embedding Problem (F. Hélein)



 $\phi \in \Gamma(\mathbb{V}^n \otimes \wedge^p \mathrm{T}^*\mathcal{M}^m)$

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Genralized Isometric Embedding Problem (F. Hélein)

$$g, \nabla \quad \mathbb{V}^{n} \underbrace{ \begin{array}{c} \Psi ? \\ } \mathcal{M}^{m} \times \mathbb{R}^{N_{m,p}^{n}} \\ d_{\nabla} \phi = 0 \\ \downarrow \\ \mathcal{M}^{m} \\ \phi \in \Gamma(\mathbb{V}^{n} \otimes \wedge^{p} \mathrm{T}^{*} \mathcal{M}^{m}) \\ \end{array}} \qquad \Psi(M, X) = (M, \Psi_{M} X)$$

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Genralized Isometric Embedding Problem (F. Hélein)

$$g, \nabla \quad \mathbb{V}^{n} \underbrace{ \begin{array}{c} \Psi ? \\ \text{Isometric} \end{array}} \qquad \mathcal{M}^{m} \times \mathbb{R}^{N_{m,p}^{n}} \\ d_{\nabla} \phi = 0 \\ \downarrow \\ \mathcal{M}^{m} \qquad \qquad \mathcal{M}^{m} \\ \phi \in \Gamma(\mathbb{V}^{n} \otimes \wedge^{p} \mathrm{T}^{*} \mathcal{M}^{m}) \qquad \Psi(M, X) = (M, \Psi_{M} X)$$

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Genralized Isometric Embedding Problem (F. Hélein)

$$g, \nabla \quad \mathbb{V}^{n} \underbrace{\Psi?}_{\text{Isometric}} \qquad \mathcal{M}^{m} \times \mathbb{R}^{N_{m,p}^{n}}$$

$$e \Gamma(\mathbb{V}^{n} \otimes \wedge^{p} \mathrm{T}^{*} \mathcal{M}^{m}) \qquad \Psi(M, X) = (M, \Psi_{M} X)$$

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Genralized Isometric Embedding Problem (F. Hélein)

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Genralized Isometric Embedding Problem (F. Hélein)

$$g, \nabla \quad \mathbb{V}^{n} \overset{\mathsf{CL}: \Psi ?}{\underset{\text{Isometric}}{}} \qquad \mathcal{M}^{m} \times \mathbb{R}^{N_{m,p}^{n}}$$

$$\overset{\mathsf{PDE}:}{\longrightarrow} \quad d\nabla \phi = 0$$

$$\overset{\mathsf{M}^{m}}{\overset{\mathsf{M}^{m}}}{\overset{\mathsf{M}^{m}}{\overset{\mathsf{M}^{m}}{\overset{\mathsf{M}^{m}}{\overset{\mathsf{M}^{m}}{\overset{\mathsf{M}^{m}}{\overset{\mathsf{M}^{m}}{\overset{\mathsf{M}^{m}}{\overset{\mathsf{M}^{m}}{\overset{\mathsf{M}^{m}}{\overset{\mathsf{M}^{m}}}{\overset{\mathsf{M}^{m}}{\overset{\mathsf{M}^{m}}}{\overset{\mathsf{M}^{m}}}{\overset{\mathsf{M}^{m}}{\overset{\mathsf{M}^{m}}}{\overset{\mathsf{M}^{m}}}{\overset{\mathsf{M}^{m}}}{\overset{\mathsf{M}^{m}}}{\overset{\mathsf{M}^{m}}}{\overset{\mathsf{M}^{m}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

• The ingredients of the generalized isometric embedding are :

$$(\mathbb{V}^n,\mathcal{M}^m,\mathsf{g},
abla,\phi)_{p}$$

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Genralized Isometric Embedding Problem (F. Hélein)

$$g_{\mathcal{N}} \bigvee^{n} \underbrace{\overset{\mathbb{C} L: \Psi ?}{\underset{\text{Isometric}}{}}}_{\text{Isometric}} \mathcal{M}^{m} \times \mathbb{R}^{N_{m,p}^{n}}$$

$$d\Psi(\phi) = 0$$

$$\int_{\mathcal{M}}^{m} \int_{\mathcal{M}}^{m} \mathcal{M}^{m}$$

$$\in \Gamma(\mathbb{V}^{n} \otimes \wedge^{p} T^{*} \mathcal{M}^{m}) \qquad \Psi(M, X) = (M, \Psi_{M} X)$$

• The ingredients of the generalized isometric embedding are :

$$(\mathbb{V}^n,\mathcal{M}^m,\mathsf{g},
abla,\phi)_p$$

• The problem is trivial when n = 1.

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Generalized Isometric Embedding Problem

What happens locally?

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Generalized Isometric Embedding Problem

What happens locally?



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Generalized Isometric Embedding Problem

What happens locally?

$$g_{,\nabla} \quad \mathbb{V}^{n} \underbrace{ \begin{array}{c} \Psi \\ \text{Isometric} \end{array}} \mathcal{M}^{m} \times \mathbb{R}^{N_{m,p}^{n}} \\ d_{\nabla} \phi = 0 \\ \downarrow \\ \lambda, \mu, \nu = 1 \text{ to } m \\ \phi \in \Gamma(\mathbb{V}^{n} \otimes \wedge^{p} \mathrm{T}^{*} \mathcal{M}^{m}) \qquad \Psi(M, X) = (M, \Psi_{M} X)$$

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Generalized Isometric Embedding Problem

What happens locally?

$$i, j, k = 1 \text{ to } n \qquad g_{i} \nabla \qquad \mathbb{V}^{n} \underbrace{ \begin{array}{c} \Psi \\ \text{Isometric} \end{array}} \mathcal{M}^{m} \times \mathbb{R}^{N_{m,p}^{n}} \quad A, B, C = 1 \text{ to } N \\ a, b, c = n + 1 \text{ to } N \\ d \nabla \phi = 0 \\ \downarrow \\ \lambda, \mu, \nu = 1 \text{ to } m \qquad \mathcal{M}^{m} \qquad \mathcal{M}^{m} \\ \phi \in \Gamma(\mathbb{V}^{n} \otimes \wedge^{p} T^{*} \mathcal{M}^{m}) \qquad \Psi(M, X) = (M, \Psi_{M} X) \\ \bullet \phi = E_{i} \phi^{i}.$$

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What happens locally?

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What happens locally?

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 $\begin{array}{cccc} i,j,k=1 \text{ to } n & g, \nabla & \mathbb{V}^n \underbrace{ \overset{\Psi}{\overset{}_{\text{Isometric}}}}_{\text{Isometric}} & \mathcal{M}^m \times \mathbb{R}^{N^n_{m,p}} & A, B, C = 1 \text{ to } N \\ & & a, b, c = n+1 \text{ to } N \\ & & d_{\nabla} \phi = 0 \\ & & & d\Psi(\phi) = 0 \\ & & & \mathcal{M}^m & & \mathcal{M}^m \end{array}$ $\Psi(M,X) = (M,\Psi_M X)$ $\phi \in \Gamma(\mathbb{V}^n \otimes \wedge^p \mathrm{T}^* \mathcal{M}^m)$ • $\phi = E_i \phi^i$. Then $d_{\nabla} \phi = 0 \Leftrightarrow d\phi^i + \eta^i_i \wedge \phi^j = 0$, where (η^i_i) is the connection 1-form of ∇ ($\nabla E_j = \eta'_i E_i$). • $\Psi(E_i) = e_i$.

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- $\phi = E_i \phi^i$. Then $d_{\nabla} \phi = 0 \Leftrightarrow d\phi^i + \eta^i_j \wedge \phi^j = 0$, where (η^i_j) is the connection 1-form of $\nabla (\nabla E_j = \eta^i_j E_i)$.
- Ψ(E_i) = e_i. Let us complement (e₁,..., e_m) to obtain an orthonormal moving frame on ℝ<sup>Nⁿ_{m,p}.
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 $i, j, k = 1 \text{ to } n \qquad g, \nabla \qquad \mathbb{V}^n \underbrace{ \stackrel{\Psi}{\underset{\text{Isometric}}{}}}_{\text{Isometric}} \mathcal{M}^m \times \mathbb{R}^{N^n_{m,p}} \quad A, B, C = 1 \text{ to } N$ $\begin{array}{c} a, b, c = n+1 \text{ to } N \\ d\Psi(\phi) = 0 \end{array}$ $\mathbf{d}_{\nabla}\,\phi = \mathbf{0}$ $\lambda, \mu, \nu = \mathbf{1} \text{ to } \mathbf{m}$ $\Psi(M,X)=(M,\Psi_MX)$ $\phi \in \Gamma(\mathbb{V}^n \otimes \wedge^p \mathrm{T}^* \mathcal{M}^m)$ • $\phi = E_i \phi^i$. Then $d_{\nabla} \phi = 0 \Leftrightarrow d\phi^i + \eta^i_i \wedge \phi^j = 0$, where (η^i_i) is the connection 1-form of ∇ ($\nabla E_j = \eta'_i E_i$). • $\Psi(E_i) = e_i$. Let us complement (e_1, \ldots, e_m) to obtain an orthonormal

moving frame on $\mathbb{R}^{N_{m,p}^n}$. Then the flat connection ω on $\mathbb{R}^{N_{m,p}^n}$ is $\omega_B^A = \langle e_A, \mathrm{d} e_B \rangle_{\mathbb{R}^{N_{m,p}^n}}$.

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What happens locally?

 $\mathbf{d}_{\nabla}\,\phi = \mathbf{0}$ $\lambda, \mu, \nu = \mathbf{1} \text{ to } \mathbf{m}$ $\Psi(M,X)=(M,\Psi_MX)$ $\phi \in \Gamma(\mathbb{V}^n \otimes \wedge^p \mathrm{T}^* \mathcal{M}^m)$ • $\phi = E_i \phi^i$. Then $d_{\nabla} \phi = 0 \Leftrightarrow d\phi^i + \eta^i_i \wedge \phi^j = 0$, where (η^i_i) is the connection 1-form of ∇ ($\nabla E_j = \eta'_i E_i$). • $\Psi(E_i) = e_i$. Let us complement (e_1, \ldots, e_m) to obtain an orthonormal moving frame on $\mathbb{R}^{N_{m,p}^n}$. Then the flat connection ω on $\mathbb{R}^{N_{m,p}^n}$ is $\omega_B^A = \langle e_A, \mathrm{d} e_B \rangle_{\mathbb{R}^{N_{m,p}^n}}.$ • $d\Psi(\phi) =$

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What happens locally?

 $i, j, k = 1 \text{ to } n \qquad g, \nabla \qquad \mathbb{V}^n \underbrace{ \stackrel{\Psi}{\longrightarrow} \mathcal{M}^m \times \mathbb{R}^{N_{m,p}^n} \quad A, B, C = 1 \text{ to } N}_{\text{Isometric}} \\ d_{\nabla} \phi = 0 \qquad \qquad \begin{vmatrix} a, b, c = n+1 \text{ to } N \\ d\Psi(\phi) = 0 \end{vmatrix}$ $\mathbf{d}_{\nabla}\,\phi = \mathbf{0}$, $\mu, \nu = \mathbf{1}$ to m , \mathcal{M}^m $\Psi(M,X)=(M,\Psi_MX)$ $\phi \in \Gamma(\mathbb{V}^n \otimes \wedge^p \mathrm{T}^* \mathcal{M}^m)$ • $\phi = E_i \phi^i$. Then $d_{\nabla} \phi = 0 \Leftrightarrow d\phi^i + \eta^i_i \wedge \phi^j = 0$, where (η^i_i) is the connection 1-form of ∇ ($\nabla E_j = \eta'_i E_i$). • $\Psi(E_i) = e_i$. Let us complement (e_1, \ldots, e_m) to obtain an orthonormal moving frame on $\mathbb{R}^{N_{m,p}^n}$. Then the flat connection ω on $\mathbb{R}^{N_{m,p}^n}$ is $\omega_B^A = \langle e_A, \mathrm{d} e_B \rangle_{\mathbb{R}^{N_{m,p}^n}}.$ • $d\Psi(\phi) = d\Psi(E_i\phi^i) =$

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What happens locally?

 $\begin{array}{c|c} i,j,k = 1 \text{ to } n & g, \nabla & \mathbb{V}^n \underbrace{ \stackrel{\Psi}{\longrightarrow} \mathcal{M}^m \times \mathbb{R}^{N_{m,p}^n} & A, B, C = 1 \text{ to } N \\ & & \\$ $\mathbf{d}_{\nabla}\,\phi = \mathbf{0}$, $\mu, \nu = \mathbf{1}$ to m , \mathcal{M}^m $\Psi(M,X)=(M,\Psi_MX)$ $\phi \in \Gamma(\mathbb{V}^n \otimes \wedge^p \mathrm{T}^* \mathcal{M}^m)$ • $\phi = E_i \phi^i$. Then $d_{\nabla} \phi = 0 \Leftrightarrow d\phi^i + \eta^i_i \wedge \phi^j = 0$, where (η^i_i) is the connection 1-form of ∇ ($\nabla E_j = \eta'_i E_i$). • $\Psi(E_i) = e_i$. Let us complement (e_1, \ldots, e_m) to obtain an orthonormal moving frame on $\mathbb{R}^{N_{m,p}^n}$. Then the flat connection ω on $\mathbb{R}^{N_{m,p}^n}$ is $\omega_{B}^{A} = \langle e_{A}, \mathrm{d} e_{B} \rangle_{\mathrm{m}N_{m}^{n}}$

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$$\mathrm{d}\Psi(\phi) = \mathrm{d}\Psi(E_i\phi^i) = \mathrm{d}(e_i\phi^i) =$$

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What happens locally?

- $\phi = E_i \phi'$. Then $d_{\nabla} \phi = 0 \Leftrightarrow d\phi' + \eta'_j \land \phi^j = 0$, where (η'_j) is the connection 1-form of $\nabla (\nabla E_j = \eta^i_j E_i)$.
- $\Psi(E_i) = e_i$. Let us complement (e_1, \ldots, e_m) to obtain an orthonormal moving frame on $\mathbb{R}^{N_{m,p}^n}$. Then the flat connection ω on $\mathbb{R}^{N_{m,p}^n}$ is $\omega_B^A = \langle e_A, \mathrm{d} e_B \rangle_{\mathbb{R}^{N_{m,p}^n}}$.

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$$\mathrm{d}\Psi(\phi) = \mathrm{d}\Psi(E_i\phi^i) = \mathrm{d}(e_i\phi^i) = e_i(\mathrm{d}\phi^i + \omega_j^i \wedge \phi^j) + e_a(\omega_i^a \wedge \phi^i) = 0.$$

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(日本)

What happens locally?

 $i, j, k = 1 \text{ to } n \qquad g_{N} \nabla \qquad \mathbb{V}^{n} \underbrace{ \stackrel{\Psi}{\underset{\text{Isometric}}{}}}_{\text{Isometric}} \mathcal{M}^{m} \times \mathbb{R}^{N_{m,p}^{n}} \quad A, B, C = 1 \text{ to } N \\ \begin{array}{c} a, b, c = n+1 \text{ to } N \\ d\Psi(\phi) = 0 \end{array}$ $\lambda, \mu,
u = 1$ to m $\phi \in \Gamma(\mathbb{V}^n \otimes \wedge^p \mathrm{T}^* \mathcal{M}^m)$ $\Psi(M,X)=(M,\Psi_MX)$ • $\phi = E_i \phi^i$. Then $d_{\nabla} \phi = 0 \Leftrightarrow d\phi^i + \eta^i_i \wedge \phi^j = 0$, where (η^i_i) is the connection 1-form of ∇ ($\nabla E_i = \eta'_i E_i$). • $\Psi(E_i) = e_i$. Let us complement (e_1, \ldots, e_m) to obtain an orthonormal moving frame on $\mathbb{R}^{N_{m,p}^n}$. Then the flat connection ω on $\mathbb{R}^{N_{m,p}^n}$ is $\omega_B^A = \langle e_A, \mathrm{d} e_B \rangle_{\mathbb{R}^{N_{m,p}^n}}.$ • $\mathrm{d}\Psi(\phi) = \mathrm{d}\Psi(E_i\phi^i) = \mathrm{d}(e_i\phi^i) = e_i(\mathrm{d}\phi^i + \omega_i^i \wedge \phi^j) + e_a(\omega_i^a \wedge \phi^i) = 0.$ $\Psi^*(\omega_i^i) = \eta_i^i$ and $\Psi^*(\omega_i^a) \wedge \phi^i = 0$

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Two Fundamental Motivations

Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

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Conservation Laws and Generalized Isometric Embeddings

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Two Fundamental Motivations

Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

 (\mathcal{M}^m,g)

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Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

$$(\mathcal{M}^m, g)$$

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Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

$$(\mathcal{M}^m, g) \subset \frac{u?}{\text{Isometric}}$$

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Two Fundamental Motivations

Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

$$(\mathcal{M}^m,g) \subset \frac{u?}{\text{Isometric}} (\mathbb{R}^N,\langle,\rangle_{\mathbb{R}^N})$$

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Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

$$(\mathcal{M}^m,g) \subset \frac{u?}{\text{Isometric}} (\mathbb{R}^N,\langle,\rangle_{\mathbb{R}^N})$$

The isometric embedding problem is equivalent to the generalized isometric embedding problem when :

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Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

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The isometric embedding problem is equivalent to the generalized isometric embedding problem when :

 $(\mathbb{V}^n, \mathcal{M}^m, g, \nabla, \phi)_p$

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Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

$$(\mathcal{M}^m,g) \subset \frac{u?}{\text{Isometric}} (\mathbb{R}^N,\langle,\rangle_{\mathbb{R}^N})$$

The isometric embedding problem is equivalent to the generalized isometric embedding problem when :

$$(\mathbb{V}^n, \mathcal{M}^m, g, \nabla, \phi)_p = (\mathrm{T}\mathcal{M}^m, \mathcal{M}^m, g,$$

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Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

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The isometric embedding problem is equivalent to the generalized isometric embedding problem when :

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and this is done through

 $\Psi(\phi) = \mathrm{d} u$

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• Cartan–Janet :

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Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

$$(\mathcal{M}^m,g) \subset \frac{u?}{\text{Isometric}} (\mathbb{R}^N,\langle,\rangle_{\mathbb{R}^N})$$

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- Cartan–Janet :
- Nash :

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Conservation Laws and Generalized Isometric Embeddings

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Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

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and this is done through

 $\Psi(\phi) = \mathrm{d} u$

- Cartan–Janet : local result
- Nash :

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Conservation Laws and Generalized Isometric Embeddings

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Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

$$(\mathcal{M}^m,g) \subset \frac{u?}{\text{Isometric}} (\mathbb{R}^N,\langle,\rangle_{\mathbb{R}^N})$$

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- Nash : global result

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Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

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- Cartan-Janet : local result in the analytic category,
- Nash : global result

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Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

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- Cartan-Janet : local result in the analytic category,
- Nash : global result in the smooth category,

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Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

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$$\Psi(\phi) = \mathrm{d} u$$

- Cartan–Janet : local result in the analytic category, N = m(m+1)/2.
- Nash : global result in the smooth category,

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Motivation 1 : Isometric Embedding Problem of Riemannian manifolds

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and this is done through

$$\Psi(\phi) = \mathrm{d} u$$

- Cartan–Janet : local result in the analytic category, N = m(m+1)/2.
- Nash : global result in the smooth category, *N* is higher.

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Two Fundamental Motivations

Motivation 2 : Harmonic maps between Riemannian manifolds

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Two Fundamental Motivations

Motivation 2 : Harmonic maps between Riemannian manifolds

 $u: (\mathcal{M}^m, g) \longrightarrow (\mathcal{N}^n, h)$ is harmonic

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Two Fundamental Motivations

Motivation 2 : Harmonic maps between Riemannian manifolds

 $u: (\mathcal{M}^m, g) \longrightarrow (\mathcal{N}^n, h)$ is harmonic if u is a critical point of the energy functional

$$\mathsf{E}[u] = \int_{\mathcal{M}^m} \frac{|\mathrm{d} u|^2}{2} \mathrm{Vol}_{\mathcal{M}^m}$$

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Two Fundamental Motivations

Motivation 2 : Harmonic maps between Riemannian manifolds

 $u: (\mathcal{M}^m, g) \longrightarrow (\mathcal{N}^n, h)$ is harmonic if u is a critical point of the energy functional

$$\mathsf{E}[u] = \int_{\mathcal{M}^m} \frac{|\mathrm{d}u|^2}{2} \mathrm{Vol}_{\mathcal{M}^m}$$

Locally, the Euler-Lagrange equations are :

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 $\Delta_g u^i$

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Locally, the Euler-Lagrange equations are :

$$\Delta_{g} u^{i} + g^{\alpha\beta} \Gamma^{i}_{jk} \left(u(x) \right) \frac{\partial u^{j}}{\partial x^{\alpha}} \frac{\partial u^{k}}{\partial x^{\beta}} = 0$$

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The identity map, constant maps,

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The identity map, constant maps, harmonic functions,

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Two Fundamental Motivations

Motivation 2 : Harmonic maps between Riemannian manifolds

 $u: (\mathcal{M}^m, g) \longrightarrow (\mathcal{N}^n, h)$ is harmonic if u is a critical point of the energy functional

$$\Xi[u] = \int_{\mathcal{M}^m} \frac{|\mathrm{d}u|^2}{2} \mathrm{Vol}_{\mathcal{M}^m}$$

Locally, the Euler-Lagrange equations are :

$$\Delta_{g} u^{i} + g^{\alpha\beta} \Gamma^{i}_{jk} \left(u(x) \right) \frac{\partial u^{j}}{\partial x^{\alpha}} \frac{\partial u^{k}}{\partial x^{\beta}} = 0$$

The identity map, constant maps, harmonic functions, parameterization of geodesics,

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Two Fundamental Motivations

Motivation 2 : Harmonic maps between Riemannian manifolds

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Locally, the Euler-Lagrange equations are :

$$\Delta_{g} u^{i} + g^{\alpha\beta} \Gamma^{i}_{jk} \left(u(x) \right) \frac{\partial u^{j}}{\partial x^{\alpha}} \frac{\partial u^{k}}{\partial x^{\beta}} = 0$$

The identity map, constant maps, harmonic functions, parameterization of geodesics, minimal isometric immersions,

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Two Fundamental Motivations

Motivation 2 : Harmonic maps between Riemannian manifolds

 $u: (\mathcal{M}^m, g) \longrightarrow (\mathcal{N}^n, h)$ is harmonic if u is a critical point of the energy functional

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The identity map, constant maps, harmonic functions, parameterization of geodesics, minimal isometric immersions, holomorphic and anti-holomorphic maps between Kählerian manifolds, ...

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Two Fundamental Motivations

Motivation 2 : Harmonic maps between Riemannian manifolds

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Two Fundamental Motivations

Motivation 2 : Harmonic maps between Riemannian manifolds

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Two Fundamental Motivations

Motivation 2 : Harmonic maps between Riemannian manifolds

$$\mathcal{M}^m \xrightarrow{u} \mathcal{N}^n$$

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Motivation 2 : Harmonic maps between Riemannian manifolds



 $d_{\nabla}(\star du) = 0 \Leftrightarrow u$ is harmonic

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Motivation 2 : Harmonic maps between Riemannian manifolds



 $d_{\nabla}(\star du) = 0 \Leftrightarrow u$ is harmonic

A harmonic map u produces the ingredients $(u^*T\mathcal{N}^n, \mathcal{M}^m, g, \nabla, \star du)_{(m-1)}$ of the generalized isometric embedding problem.

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The Keystone!

Two Fundamental Motivations

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Two Fundamental Motivations

The Keystone! Noether's Theorem!

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Two Fundamental Motivations

The Keystone! Noether's Theorem!



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The Keystone! Noether's Theorem!

$$\begin{array}{c} g, \nabla, u^* T \mathcal{S}^n \\ d_{\nabla}(\star du) \\ \mathcal{M}^m \end{array}$$

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The Keystone! Noether's Theorem!

$$\begin{array}{c} g, \nabla, u^* T \mathcal{S}^n & \smile \Psi \\ \\ d_{\nabla}(\star du) \\ & \downarrow \\ \mathcal{M}^m \end{array}$$

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Two Fundamental Motivations

The Keystone! Noether's Theorem!



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The Keystone! Noether's Theorem!



 $\Psi(M,v)=(M,u\times v)$

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Two Fundamental Motivations

The Generalized Isometric Embedding Problem's Goal

To show the existence of the analogeous of conservation laws when there are no symmetries for a system of partial differential equations.

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Results

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Results

Construction of local conservation laws by generalized isometric embedding of vector bundles, arXiv :0804.2608 (2010), to appear in Asian J. of Math.

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Results

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Theorem $((\mathbb{V}^n, \mathcal{M}^m, \mathbf{g}, \nabla, \phi)_{m-1} \text{ case})$

Let \mathbb{V}^n be a real analytic n-dimensional vector bundle over a real analytic *m*-dimensional manifold \mathcal{M}^m endowed with a metric *g* and a connection ∇ compatible with *g*. Given a non-vanishing covariantly closed \mathbb{V}^n -valued differential (m-1)-form ϕ , there exists a local generalized isometric embedding of \mathbb{V}^n in $\mathcal{M}^m \times \mathbb{R}^{n+\kappa_{m,m-1}^n}$ over \mathcal{M}^m , where $\kappa_{m,m-1}^n \ge (m-1)(n-1)$ such that the image of ϕ is a conservation law.

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Corollary

Let (\mathcal{M}^m, g) be a real analytic m-dimensional Riemannian manifold,

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Corollary

Let (\mathcal{M}^m, g) be a real analytic m-dimensional Riemannian manifold, ∇ be the Levi-Civita connection

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Corollary

Let (\mathcal{M}^m, g) be a real analytic m-dimensional Riemannian manifold, ∇ be the Levi-Civita connectionand T be a contravariant 2-tensor with a vanishing covariant divergence.

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Corollary

Let (\mathcal{M}^m, g) be a real analytic m-dimensional Riemannian manifold, ∇ be the Levi-Civita connectionand T be a contravariant 2-tensor with a vanishing covariant divergence. Then there exists a local conservation law for T on $\mathcal{M}^m \times \mathbb{R}^{m+(m-1)^2}$.

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Application : m = 4 and T the stress-energy tensor.

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Application : m = 4 and T the stress-energy tensor.

$$\Gamma(\mathrm{T}\mathcal{M}^m\otimes\mathrm{T}\mathcal{M}^m)\longrightarrow\Gamma(\mathrm{T}\mathcal{M}^m\otimes\wedge^{m-1}\mathrm{T}^*\mathcal{M}^m)$$
$$T\mapsto\tau$$

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$$\Gamma(\mathrm{T}\mathcal{M}^m\otimes\mathrm{T}\mathcal{M}^m)\longrightarrow \Gamma(\mathrm{T}\mathcal{M}^m\otimes\wedge^{m-1}\mathrm{T}^*\mathcal{M}^m)$$
$$T\mapsto \tau$$

$$\operatorname{div} T = \mathbf{0} \Longleftrightarrow \operatorname{d}_{\nabla} \tau = \mathbf{0}$$

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Results

Results

Theorem (($\mathbb{V}^2, \mathcal{M}^m, \mathbf{g}, \nabla, \phi$)₁ case)

Let \mathbb{V}^2 be a real analytic 2-dimensional vector bundle over a real analytic *m*-dimensional manifold \mathcal{M}^m endowed with a metric *g* and a connection ∇ compatible with *g*. Given a non-vanishing covariantly closed non-degenerate \mathbb{V}^2 -valued differential 1-form ϕ , there exists a local generalized isometric embedding of \mathbb{V}^2 in $\mathcal{M}^m \times \mathbb{R}^{n+\kappa_{m,1}^2}$ over \mathcal{M}^m , where $\kappa_{m,m-1}^n \ge 1$ such that the image of ϕ is a conservation law.

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Results

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Results

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Results

Results

Theorem (($\mathbb{V}^3, \mathcal{M}^4, \mathbf{g}, \nabla, \phi$)₂ ASD case)

Let \mathcal{M}^4 be an oriented real analytic 4-dimensional manifold endowed with a metric (actually a conformal structure is enough). Consider a real analytic vector bundle \mathbb{V}^3 of rank 3 over \mathcal{M}^4 , endowed with a Riemannian metric g, g-compatible connection ∇ , and a anti-self-dual covariantly closed \mathbb{V}^3 -valued differential 2-form ϕ . There exists then a local generalized isometric embedding Ψ of \mathbb{V}^3 into $\mathcal{M}^4 \times \mathbb{R}^{3+\kappa^3_{4,2,ASD}}$, where $\kappa^3_{4,2,ASD} \ge 4$, such that $\Psi(\phi)$ is a local conservation law.

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Results

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Results

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Frédéric Hélein, *Manifolds obtained by soldering together points, lines, etc.* in Geometry, topology, quantum field theory and cosmology, C. Barbachoux, J. Kouneiher, F. Hélein, eds, collection Travaux en Cours (Physique-Mathématiques), Hermann 2009, p. 23–43. **arXiv :0904.4616.**

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Solving Strategy

Solving Strategy

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Solving Strategy

Solving Strategy

$$\Psi^*(\omega^i_j) = \eta^i_j$$
 and $\Psi^*(\omega^a_i) \wedge \phi^i = 0$

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Solving Strategy

Solving Strategy

$$\Psi^*(\omega^i_j)=\eta^i_j$$
 and $\Psi^*(\omega^a_i)\wedge\phi^i=0$

On the product manifold

$$\Sigma_{m,p}^{n} = \mathcal{M}^{m} imes rac{SO(n + \kappa_{m,p}^{n})}{SO(\kappa_{m,p}^{n})}$$

the naive EDS is

$$\mathcal{I}_{m,p}^{n} = \{\omega_{j}^{i} - \eta_{j}^{i}, \omega_{i}^{a} \wedge \phi^{i}\}$$

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The generalized isometric embedding EDS is

$$\mathcal{I}_{m,p}^{n} = \{\omega_{j}^{i} - \eta_{j}^{i}, \omega_{a}^{i} \wedge \omega_{j}^{a} + \Omega_{j}^{i}, \omega_{i}^{a} \wedge \phi^{i}\}$$

On the product manifold

$$\Sigma_{m,p}^{n} = \mathcal{M}^{m} \times \frac{SO(n + \kappa_{m,p}^{n})}{SO(\kappa_{m,p}^{n})}$$

where Ω is the curvature 2-form of $\nabla (\Omega_j^i = \mathrm{d}\eta_j^i + \eta_k^i \wedge \eta_j^k)$.

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Solving Strategy

Solving Strategy

Solving Strategy

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Solving Strategy

Solving Strategy

The standard case is $(\mathrm{T}\mathcal{M}^{\textit{m}},\mathcal{M}^{\textit{m}},g,\nabla,\mathrm{Id}_{\mathrm{T}\mathcal{M}^{\textit{m}}})_{1}$

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 Solving Strategy

The standard case is $(\mathrm{T}\mathcal{M}^{\textit{m}},\mathcal{M}^{\textit{m}},g,\nabla,\mathrm{Id}_{\mathrm{T}\mathcal{M}^{\textit{m}}})_{1}$

• Generalized torsion :

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Solving Strategy

The standard case is $(\mathrm{T}\mathcal{M}^{\textit{m}},\mathcal{M}^{\textit{m}},g,\nabla,\mathrm{Id}_{\mathrm{T}\mathcal{M}^{\textit{m}}})_{1}$

• Generalized torsion : $\Theta = E_i \Theta^i := E_i (d\phi^i + \eta^i_i \wedge \phi^j).$

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Solving Strategy

The standard case is $(\mathrm{T}\mathcal{M}^{\textit{m}},\mathcal{M}^{\textit{m}},g,\nabla,\mathrm{Id}_{\mathrm{T}\mathcal{M}^{\textit{m}}})_{1}$

- Generalized torsion : $\Theta = E_i \Theta^i := E_i (d\phi^i + \eta^i_j \wedge \phi^j).$
- Generalized Bianchi identities :

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Solving Strategy

The standard case is $(\mathrm{T}\mathcal{M}^{\textit{m}},\mathcal{M}^{\textit{m}},g,\nabla,\mathrm{Id}_{\mathrm{T}\mathcal{M}^{\textit{m}}})_1$

- Generalized torsion : $\Theta = E_i \Theta^i := E_i (d\phi^i + \eta^i_j \wedge \phi^j).$
- Generalized Bianchi identities : $\mathcal{B}_{m,p}^{n} := E_{i}(\Omega_{j}^{i} \wedge \phi^{j}) = 0.$

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Solving Strategy

The standard case is $(\mathrm{T}\mathcal{M}^{\textit{m}},\mathcal{M}^{\textit{m}},g,\nabla,\mathrm{Id}_{\mathrm{T}\mathcal{M}^{\textit{m}}})_{1}$

- Generalized torsion : $\Theta = E_i \Theta^i := E_i (d\phi^i + \eta^i_j \wedge \phi^j).$
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$$\mathcal{K}^n_{m,p} := \{(\mathcal{R}^i_{j;\lambda\mu}) \in \wedge^2(\mathbb{R}^n) \otimes \wedge^2(\mathbb{R}^m) | \ \Omega^i_j \wedge \phi^j = 0, \forall i = 1 \ ext{to} \ n\}.$$

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Solving Strategy

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Solving Strategy

Solving Strategy

The prolongation process of $\mathcal{I}_{m,p}^n$ yields to :

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Solving Strategy

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The prolongation process of $\mathcal{I}^n_{m,p}$ yields to : $\omega^a_i=H^a_{i\lambda}\eta^\lambda$

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Solving Strategy

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The prolongation process of $\mathcal{I}_{m,p}^n$ yields to : $\omega_i^a = H_{i\lambda}^a \eta^\lambda$ where $H_{i\lambda}^a \in \mathcal{W}_{m,m-1}^n \otimes \mathbb{R}^n \otimes \mathbb{R}^m$

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Solving Strategy

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Solving Strategy

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• Generalized Cartan Identities :

$$\sum_{\substack{\lambda = 1, \dots, m \\ 1 \leqslant \mu_1 < \dots < \mu_p \leqslant m}} H^a_{i\lambda} \psi^i_{\mu_1, \dots, \mu_p} = 0$$

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• Generalized Gauss Map :

$$(\mathcal{G}_{m,p}^n)_{j;\lambda\mu}^i:H_{i\lambda}.H_{j\mu}-H_{i\mu}.H_{j\lambda}=\mathcal{R}_{j;\lambda\mu}^i.$$

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Solving Strategy

Specialization For The Conservation Law Case : p = m - 1

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Lemma

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Lemma

Let $\kappa_{m,m-1}^n \ge (m-1)(n-1)$.

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Specialization For The Conservation Law Case : p = m - 1

Lemma

Let $\kappa_{m,m-1}^n \ge (m-1)(n-1)$. Let $\mathcal{H}_{m,m-1}^n(M) \subset \mathcal{W}_{m,m-1}^n \otimes \mathbb{R}^n \otimes \mathbb{R}^m$ be the open set consisting of those elements $H = (H_{i\lambda}^a)$ so that the vectors $\{H_{i\lambda} | i = 1, \dots, n-1 \text{ and } \lambda = 1, \dots, m-1\}$ are linearly independent as elements of $\mathcal{W}_{m,m-1}^n$

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Solving Strategy

Specialization For The Conservation Law Case : p = m - 1

Lemma

Let $\kappa_{m,m-1}^n \ge (m-1)(n-1)$. Let $\mathcal{H}_{m,m-1}^n(M) \subset \mathcal{W}_{m,m-1}^n \otimes \mathbb{R}^n \otimes \mathbb{R}^m$ be the open set consisting of those elements $H = (H_{i\lambda}^a)$ so that the vectors $\{H_{i\lambda} | i = 1, ..., n-1 \text{ and } \lambda = 1, ..., m-1\}$ are linearly independent as elements of $\mathcal{W}_{m,m-1}^n$ and satisfy the generalized Cartan identities. Then $\mathcal{G}_{m,m-1}^n : \mathcal{H}_{m,m-1}^n \longrightarrow \mathcal{K}_{m,m-1}^n$

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Solving Strategy

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Lemma

Let $\kappa_{m,m-1}^n \ge (m-1)(n-1)$. Let $\mathcal{H}_{m,m-1}^n(M) \subset \mathcal{W}_{m,m-1}^n \otimes \mathbb{R}^n \otimes \mathbb{R}^m$ be the open set consisting of those elements $H = (H_{i\lambda}^a)$ so that the vectors $\{H_{i\lambda} | i = 1, ..., n-1 \text{ and } \lambda = 1, ..., m-1\}$ are linearly independent as elements of $\mathcal{W}_{m,m-1}^n$ and satisfy the generalized Cartan identities. Then $\mathcal{G}_{m,m-1}^n : \mathcal{H}_{m,m-1}^n \longrightarrow \mathcal{K}_{m,m-1}^n$ is a surjective submersion.

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Perspectives

Perspectives

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Perspectives

Perspectives

Global versions in the analytic category.

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Perspectives

Perspectives

- Global versions in the analytic category.
- ② Local and/or global versions in the smooth category.

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- Global versions in the analytic category.
- Output Local and/or global versions in the smooth category.
- Solving the problem for $p = 1, \ldots, m 2$.

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- Global versions in the analytic category.
- Output Description of the second s
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- Studying the rigidity of the generalized isometric immersions.
- Studying the problem for a structural group $G \subset O(n)$.

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