Graduate Calculus Curriculum Review, Spring 2007

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A copy of this document may be found at
http://www.math.duke.edu/∼adsmith/proposal.pdf

Abstract. This document is the end result of a several-month-long study of the calculus curriculum by a group of experienced graduate instructors. Herein, we propose specific changes to each course as well as significant changes to the curriculum as a whole. We arrived at these proposals after carefully considering the mathematical content of each class, its intended audience, and the mathematical requirements of the various physical, biological, and social sciences. In the course of gathering this information, we interviewed graduate instructors, teaching faculty, and teachers of concurrent courses in many other departments.

For each of 25L, 26L, 31L, 32L and the multivariable courses, we suggest shifting the position or emphasis of various topics, and we suggest alterations to specific lab assignments. In several cases, we have replacement labs in concept or in outline, and expect to write the new labs during Spring and Summer.

Most significantly, we propose replacing 32 and 41 with 41L, a course we designed specifically for incoming freshmen who have gained AP credit for 31. The course is a careful balance of rigorous mathematics, computational techniques, and physical applications.

Contents

CHAPTER 1

Summary

In the current curriculum, freshmen enter into any of Math 25L, 31L, 32L, 32, or multivariable calculus, depending upon their AP Calculus and SAT scores. Figure 1 indicates the current system.

This setup leaves a lot to be desired, especially at the Calculus II level. Teaching 32L is a challenge because the class contains two entirely different populations of students: the mathematically gifted freshmen, and the mathematically challenged upperclassmen, many of these coming from the 25L/26L sequence. The instructor is faced with the difficult task of challenging the freshmen while not alienating the upperclassmen. In addition to the disparity among students' skill levels, there is a disparity of backgrounds in 32L. One striking example of this disparity concerns differential equations. Upperclassmen have been exposed to the basics of differential equations in 25L and 26L, while many freshmen have not encountered them before. Math 32L assumes a prior knowledge of differential equations, so no introduction is included. Instructors are forced to take class time to fill in the gaps in freshmen's backgrounds, while upperclassmen nod off.

In addition to the problems facing Math 32L, we feel that the students' needs in Math 32 are currently not being met. The course is not producing math majors; instead, the vast variety of students taking 32 will major in Chemistry, Biology, Engineering, Economics, and Physics, to name a few. Currently, 32 has very little differential equations content, even though this is one of the main ways that other disciplines use calculus. Instead, students in 32 are inundated with many, many techniques of integration despite the fact that representatives from other departments insist that they only require their students to know the

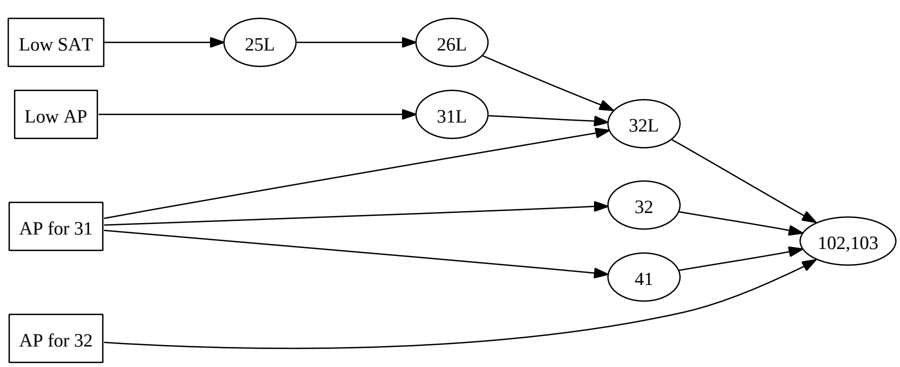

FIGURE 1. A flowchart of the current curriculum where "Low SAT" denotes a score of below 680 on Math SAT or the Math SAT II, "Low AP" denotes a 1-4 on the AB or a 1-2 on the BC AP tests, "AP for 31" denotes a score of 5 on the AB test or a 3 on the BC test, and "AP for 32" denotes a score of 4 or 5 on the BC test.

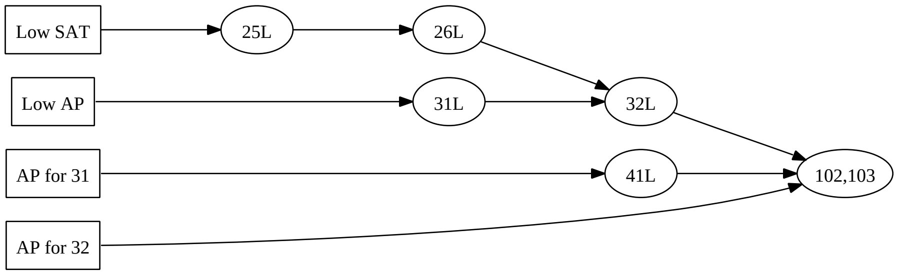

FIGURE 2. Flowchart of the proposed curriculum, where placement is as in Figure 1.

basic integration techniques and formulas. They would much rather that their students be able to solve real-world problems, and to understand how the mathematics describes the physical world. Currently, there are only two "applications" in Math 32: the work involved in filling oddly shaped tanks, and a brief introduction to separable differential equations including examples like Newton's Law of Cooling. There are many more interesting examples that could be integrated into the course to show students how mathematics is used and to connect the mathematics to the students' interests in a concrete way. Finally, one of the stated goals of Math 32 is for students to "develop an understanding and appreciation of the formal structure of calculus," according to the department web-page. However, the vast majority of the homework problems, and nearly all of the problems on recent final exams, are lengthy computations. In order to get our students to understand the rigor of calculus, we must give the students adequate practice, and we must assess it on tests and the final exam.

To fix these problems and others discussed at length in this report, we propose that 32 evolve into a new lab course called 41L. This would be a course only for students coming to Duke with AP credit for Calculus I. The proposal is represented in Figure 2.

Under the proposal, since all students with AP credit for Calculus I will take 41L, some of the disparity issues in 32L are immediately resolved. The new course should have a laboratory component so that students have the opportunity to explore difficult exercises, examples, and applications. The new course would have many fewer techniques of integration, but students would have to pass a Gateway competency test in integration in order to pass the class. Also, the new course would have many interesting applications from other disciplines to connect the mathematics to the real world. An entire section on differential equations will introduce freshmen to many different types of differential equations and will focus on how mathematics can be used to model a variety of situations. In addition, the course will approach calculus from a very rigorous point of view to help better prepare students to take multivariable calculus. The definite integral will be carefully defined, and the Fundamental Theorem of Calculus will be proven using the Mean Value Theorem. Also, series, Taylor series, and Fourier series will be covered carefully, including the ε-N definition of the limit of a sequence. A fairly complete but rough version of the syllabus and our choice of textbook is given later in the report.

In addition to our main proposal to evolve 32 into this new class 41L, we have suggestions for improving other courses. For 25L/26L, the Euler's Method Lab does not sufficiently convey the idea, the order of topics needs further thought, and a more structured syllabus is needed because of "textbook hopping." For 31L, we are dissatisfied with the amount of time and energy spent on the method called z-substitution. We conclude that an in-depth discussion in the instructor's syllabus to warn teachers of potential hazards and discuss alternatives would go a long way toward alleviating this problem. For 32L, certain topics are rushed (like the phase-plane material) while others are dwelled upon (like discrete probability). In addition, Taylor polynomials seem unmotivated, and there are some very confusing labs. We recommend some minor syllabus changes and lab rewrites to improve the course for the students. The main change recommended for 103 is the removal of some subjects to make the class run at a more manageable pace.

Graduate students teach a majority of the lower-level Calculus courses here at Duke and have accumulated much valuable experience teaching both laboratory courses and non-laboratory courses. In this report we make several administrative recommendations regarding the role of graduate students in the curriculum and aspects of the curriculum oversight process. We hope that this report will encourage more communication among *all* instructors here at Duke and also hope that graduate students will continue to be involved in the process of curriculum review in a formal way. We invite and encourage any feedback or questions.

CHAPTER 2

The Role of Labs

1. Summary

The role of labs in calculus courses has evolved significantly since the days of Project CALC and has now achieved a beneficial symbiotic role with traditional lectures. In our experience labs have been a valuable tool for single variable calculus courses and can improve student learning in any such course.

2. Discussion

In addition to three 50-minutes lectures, lab courses have a weekly 105-minute session, during which students work in groups to investigate informative exercises or physical examples relating to the current topic. The students are often assigned a written report on this lab material, and their knowledge is reinforced and verified by quizzes and exam questions.

Because of the history of calculus reform at Duke, people sometimes conflate the current lab-based curriculum with Project CALC. This is an error. Any course, no matter how "formal" or "conceptual," can benefit from a lab component. Labs do not replace prepared lectures; instead, they offer another effective method of teaching students through means not provided by traditional lectures or homework alone.

Labs are an opportunity to explore difficult exercises, examples and applications. Usually, instructors cannot hope that students have mulled over a difficult concept for an hour or two, but the labs guarantee that students discuss and ponder challenging problems.

Also, many mathematical ideas are built upon motivating examples. Labs force the students to study these examples on their own concurrently with lectures on the topic, and this self-discovery seems to give the students confidence and a sense of ownership for the material.

Finally, labs allow for student-teacher contact outside of the standard one-directional lecture or desperate office-hour scenarios, and this allows for more flexible scheduling if a class needs special attention on a certain topic.

All of the teachers we interviewed from other departments insisted that the greatest lack in their students was not computational ability; rather, their students have difficulty first recognizing when a physical problem can be expressed mathematically and then applying mathematics to solve the physical problem. The labs which apply calculus techniques to physical problems directly address this concern.

In our judgment, the only downside of lab courses is the expense of additional staff to operate the labs. Unfortunately, some lab instructors and assistants are not fully competent to help students, and others do not have sufficient mastery of analysis to give insightful answers to confused students. Unreliable lab instruction makes the lecturer's job more difficult, as lecture time is absorbed by clarification of lab material. Hopefully, these problems can be addressed by careful selection of lab assistants and more emphasis upon communication, including mandatory meetings among the teachers and lab assistants for each course.

CHAPTER 3

25L & 26L: Introductory Calculus

1. Summary

The committee's assessment of 25L and 26L is that these courses are fundamentally sound and generally meet the needs of their students. With this in mind, we did find a few areas where the course suffers and have made some suggestions.

2. Problems and Solutions

The 25/26L course is an introductory calculus course with pre-calculus review spread throughout the course. It targets the students who do not have the background for 31L or higher.

2.1. Problems.

- **Student backgrounds.** Students in 25L have a wide range of mathematical backgrounds. While the majority of students are well placed, some students should be in 31L and are taking 25L for the easier grade or due to low self evaluation. Other students do not have the necessary prerequisite math skills to succeed in 25L.
- **Textbook Hopping.** In order to introduce calculus early on and to provide a mix of calculus and pre-calculus topics throughout the course, the syllabus jumps around in the textbook. Thus, because they have not seen the relevant material, students are not prepared to work through many of the examples in the text. For example, students do not see trig functions until 26L, but many examples in the reading for 25L are trig based.
- **Order of Topics.** Math 26L covers differential equations before covering integration or anti-differentiation. The textbook's presentation of separation of variables relies heavily on integration, which students have not seen at this point in the course.
- **Euler's Method Lab.** Euler's method could be valuable as an example of linear approximation. The current lab, however, does not sufficiently convey the idea. Many students come away understanding Euler's method to be an exercise in filling in tables. In addition, the lab in its current format allows only one basic type of test question, which has appeared on all past finals in recent memory.

2.2. Possible Solutions.

- **Student Backgrounds.** There seems to be no obvious solution to the background problems. Standardized test scores or a very early assessment of pre-calculus skills could possibly identify stronger students for transfer into 31L. For the weaker students, the traditional solution has been a pre-calculus course. However, we agree with the decision not to offer one, especially since these students seem to primarily lack algebra (not pre-calculus) skills. The course does cover pre-calculus ideas as needed.

- **Textbook Hopping.** The direct solution of choosing a different textbook is overkill. The benefit of using the same textbook as 31L and 32L is significant, and this issue is a minor one. A simple measure would be to provide the students with a more detailed syllabus telling them which sections of their readings should be skipped.
- **Order of Topics.** The solution of covering integration before differential equations seems natural. Of course, it is not an accident that the course is taught in this order. The class was designed in this way to emphasize that ODEs are a differential calculus topic. There is a trade-off here that needs to be considered.
- **Euler's Method Lab.** We suggest reworking the current lab or writing a new lab to emphasize the idea of linear approximation more and to reduce the focus on the table algorithm. Mike Nicholas has briefly discussed a new lab with Jim Tomberg, and a suggested first draft has been written.

CHAPTER 4

31L: Calculus I

1. Summary

We think that Math 31L is a very good course. We recommend one small change to the instructor's version of the syllabus so that first-time teachers are fully aware of later topics.

2. Discussion

Math 31L is, for the most part, a great course. The labs merge seamlessly into the rest of the course, and all topics are very appropriate. We are dissatisfied with one aspect of the course, and that is z-substitution. This is a method for solving differential equations of the form $y' = ay + b$.

The issue is that this method only works for this very specific differential equation. Many students spend a lot of time and energy mastering this technique, only for it to become largely obsolete when the method of separation of variables is covered later in the course. It would be nice if that time and energy could be directed to more worthwhile endeavors. Also, as is often the case, students often memorize the algorithm without understanding any part of the bigger picture.

We looked at the syllabus long and hard and did not see any nice way to remedy this situation. Our understanding is that this technique is emphasized because it allows more varied differential equations to be discussed before integration is introduced. This, some argue, completely justifies this technique being included in this course. Removal of this topic is impossible without overhauling the entire course.

We suggest that the instructor's version of the syllabus include a discussion of why z-substitution is emphasized and why the order of topics is the way that it is. Also, such a note would inform first-time teachers of 31L that students will learn separation of variables later in the course. Some teachers may want to give students a preview of separation of variables, and some may want to spend less class time on z-substitution. Furthermore, instructors can warn students that a more general method of solving some differential equations is coming. This would help to ease both teacher and student frustration. The important thing is that well-informed teachers and students can make better decisions.

We think that informing teachers and students about the benefits and potential hazards of z-substitution is probably the best solution to this problem. Some of us are still dissatisfied, but until a better alternative is found, we will stand behind this suggestion.

CHAPTER 5

32L: Calculus II

1. Summary

32L is the integral calculus course primarily directed towards those students who have previously taken a lab-based differential calculus course at Duke. Because the fundamental theorem is discussed in 26L and 31L, this course focuses on applications of integrals and series. This makes the course inappropriate for freshmen entering with AP credit for 31, and we intend to place such students in 41L, described later. Additionally, several syllabus changes and lab rewrites can streamline the course for its intended audience.

2. Problems and Solutions

Currently, the most significant problem with 32L is the bi-modal grade distribution which teachers have frequently observed, particularly during the Fall semester. Incoming freshmen who obtain AP credit for 31 compete with students who have just finished 25L and 26L, courses which operate at a slower pace than 32L. Professor Bookman has obtained GPA data which suggests that the incoming freshmen outperform the sophomores by a large margin. (See Table 2 in Appendix A.) However, the incoming freshmen may never see a complete treatment of the Fundamental Theorem or the careful exposure to differential equations which occurs in 25L, 26L, and 31L.

By separating the freshmen into 41L (as described in Chapters 6 and 7), their needs can be addressed and their skills can be challenged more appropriately. Through this change 32L can be optimized towards those students who have just finished 26L or 31L, where we know they have gained a clear understanding of derivatives, the fundamental theorem, and basic separable differential equations.

Additionally, it is important to note that this course varies from Fall to Spring. In the Fall most students are coming from 26L (assuming we send freshmen to 41L). In the Spring most students are coming from 31L.

Based on our discussions and interviews, most teachers are generally pleased with the topics presented in the current 32L syllabus, but there are some syllabus alterations which could improve the flow of the course.

2.1. Syllabus Problems.

- **Topics Jumbled.** During the Fall semester, the ratio test and alternating series tests are discussed 5 weeks after the other series tests. This confuses students, as they already have studied and been tested on many series convergence tests, but new tests suddenly appear. The probability material is also spread between weeks 1 and 7.

- **Certain Topics Rushed.** The difficult phase-plane material at the end of the course is often rushed, while the discrete probability at the beginning of the course is often dwelled upon even though students grasp it fairly quickly.
- **Repeated Topics.** Some topics are repeated from 25L, 26L, and 31L. In the current Fall syllabus, this is unavoidable, as incoming freshmen have never seen some topics. In particular, the time-consuming Normal Datasets lab is repeated by students coming from 26L.
- **Taylor Polynomials Unmotivated.** Though most teachers approach Taylor polynomials as local approximations to functions, this perspective is not stressed in the syllabus, thus its emphasis in the course is only folkloric.
- **Fourier Series Notation.** Hughes-Hallett uses non-standard and confusing notation for Fourier series.
- **Confusing Labs.** The Present and Future Value lab and the Air Pollution lab consistently cause complaints from both students and teachers. The PV-FV lab is ambiguous about the meaning of Present Value, and it confuses inflation with earned interest. The Air Pollution lab has a very contrived measurement (number of particles per size), which makes the integration problem senseless. This lab also has placement problems. During the Fall semester, it takes place in the first week—long before any integration or Riemann sums have been studied. During the Spring semester, it is better placed, but it would be better placed after still more physical examples of integration.

2.2. Syllabus Solutions.

- **Topics Jumbled.** The ratio test and alternating series tests should always be presented in weeks 3 and 4 with the other series tests. Because of the necessity of integration techniques for continuous probability, that topic should not be moved, as it is located to motivate integration techniques.
- **Certain Topics Rushed.** Students seem to grasp the discrete probability material just fine in the Spring, when only two days are devoted to it. In the current Fall schedule, four days is too much. No more than three days should be devoted to discrete probability, and two days are probably sufficient.
- **Repeated Topics.** Once the freshmen are moved to 41L, we can remove the repeated topics. In particular, the Normal Datasets lab should be removed in the Fall syllabus. Also, Force and Work are never addressed in the context of integration in 32L, and this seems to be a significant lack.
- **Taylor Polynomials Unmotivated.** A lab should be written which uses Taylor polynomials to approximate interesting but elusive functions, such as e^{-x^2}. One idea is to use a 4th degree Taylor approximation to study the normal distribution. Additionally, when a specific example function is used, the Taylor error can be examined in a direct way, making it accessible without having to fully derive the Taylor error formula in the general case.
- **Fourier Series Notation.** We could write our own section on Fourier series without too much difficulty, as half of the material is already mentioned in the lab. Alternatively, the book we recommend for 41L has a good on-line section on Fourier series which may be usable for both 41L and 32L, copyright permitting.

- **Confusing Labs.** The PV-FV lab should be rewritten or removed, and the Air Pollution lab should be moved to week 7.

3. Syllabus

Here is a slightly revised 32L syllabus. Because of the slight difference in background between the 26L students in the Fall and 31L students in the Spring, the syllabus must vary slightly between semesters.

3.1. Fall Syllabus. This syllabus is written to match the Fall 2006 calendar for easy comparison with the current syllabus.

1-1 Probability: Events
1-2 Probability: Random Variables and Expected Values
Lab: Probability and Geometric Series
1-3 Geometric Series

2-1 Sequences and Partial Sums
2-2 n-th Term Test
Lab: Integrating to Infinity
2-3 Integral Test

3-1 Absolute Convergence, Comparison and Limit Comparison Tests
3-2 Ratio Test
Lab: Series worksheet
3-3 Alternating Series Test

4-1 review day
4-2 review day
Lab: Test #1
4-3 Integration by Substitution

5-1 Approximating Definite Integrals
5-2 Integration by Parts
Lab: Force and Work, **NEW**
5-3 Integration by Algebraic Identities

6-1 Improper Integrals
6-2 Distribution Functions
Lab: Gateway Test
6-3 Continuous Probability Distributions

7-1 (Fall break)
7-2 Normal Distributions
Lab: Air Pollution: Fine Particulate Matter, **REWORK**
7-3 Areas and Volumes

8-1 Volume and Arc-length
8-2 Taylor Polynomials
Lab: Approximation with Taylor Polynomials, **NEW**
8-3 Taylor Polynomials

9-1 review day
9-2 review day
Lab: Test #2
9-3 Convergence of Power Series

10-1 Taylor Series
10-2 Using Taylor Series
Lab: Series Solutions to Initial Value Problems
10-3 Using Taylor Series

11-1 Fourier Series with period 2π, **NEW TEXT**
11-2 Fourier Series review
Lab: Fourier Analysis of Musical Sound
11-3 Fourier Series with general period

12-3 Fourier Series review
12-3 Oscillations
Lab: Limited Immunity in Epidemics
12-3 Predator Prey with Phase Planes

13-1 SIR model with Phase Plane
13-2 (Thanksgiving holiday)
13-3 (Thanksgiving holiday)

14-1 Phase Plane Analysis
14-2 Oscillations with Phase Plane
Lab: Gateway makeup
14-3 review

15-1 review
15-2 Test #3
15-3 reviews and TCE

3.2. Spring Syllabus. This syllabus is written to match the Spring 2007 calendar for easy comparison with the current syllabus.

1-1 (no day)
1-2 Probability: Events
1-3 Probability: Random Variables and Expected Values

2-1 (holiday)

Lab: Probability and Geometric Series
2-2 Geometric Series
2-3 Sequences and Partial Sums

3-1 n-th Term Test
Lab: Integrating to Infinity
3-2 Integral Test
3-3 Absolute Convergence, Comparison and Limit Comparison Tests

4-1 Ratio Test
Lab: Series worksheet and Normal Datasets I
4-2 Alternating Series Test
4-3 review day

5-1 review day
Lab: Test #1
5-2 Integration by Substitution
5-3 Approximating Definite Integrals

6-1 Integration by Parts
Lab: Force and Work, **NEW**
6-2 Integration by Algebraic Identities
6-3 Improper Integrals

7-1 Distribution Functions
Lab: Gateway Test
7-2 Continuous Probability Distributions
7-3 Normal Distributions

8-1 Areas and Volumes
Lab: Normal Datasets Lab, part II
8-2 Volume and Arc-length
8-3 review

9-1 Taylor Polynomials
Lab: Approximation with Taylor Polynomials, **NEW**
9-2 Taylor Polynomials
9-3 Convergence of Power Series

10 spring break

11-1 review day
Lab: Test #2
11-2 Taylor Series
11-3 Using Taylor Series

12-1 Using Taylor Series
Lab: Series Solutions to Initial Value Problems
12-2 Fourier Series with period 2π, **NEW TEXT**
12-3 Fourier Series review

13-1 Fourier Series with general period
Lab: Fourier Analysis of Musical Sound
13-2 Fourier Series review
13-3 Oscillations

14-1 Predator Prey with Phase Planes
Lab: Limited Immunity in Epidemics
14-2 SIR model with Phase Plane
14-3 Phase Plane Analysis

15-1 Oscillations with Phase Plane
Lab: Gateway makeup
15-2 review
15-3 review

16-1 Test #3
16-2 reviews and TCE

CHAPTER 6

32: Calculus II for Freshmen with AP Credit

1. Summary

Currently, freshmen who have scored a 5 on the AB Calculus test or a 3 on the BC Calculus test are given the choice to enroll in either 32 or 32L. As previously discussed, 32L is inappropriate for these students because of discrepancies in background. Because of its schedule, lack of rigor, excessive computation, and monotonous applications, 32 has collected numerous complaints from instructors and students over the past several years.

In this chapter, we outline these fundamental problems with 32. We suggest that the solution is to replace 32 with a new course, 41L, which is designed specifically for incoming freshmen and which directly address their deficiencies of rigor and application.

2. Problems and Solutions

This class is intended for freshmen who took calculus in high-school and earned credit for 31, and it is suggested over 32L to students who are interested in mathematics and related fields. However, it does not appear to produce many mathematics majors, as shown in Table 3 in Appendix A. This strongly indicates that this course is not serving its intended audience. We believe that this is true for the following reasons:

- **Excessively Esoteric Computation.** Most of the students entering Duke with credit for 31 are reasonably competent at computation since most of high-school mathematics is treated as algorithmic symbol manipulation (particularly on the AP test). The emphasis on pencil-and-paper computation in 32 is therefore a comfortable mode of learning to the students, and they are relatively unfazed by page-long integral calculations. Also, many of the integral techniques are already required on the AP exam, which all of these students have passed. Moreover, the harder integrals are completely unmotivated, making it clear to the students that these techniques are merely to be memorized for the exam.

  The faculty from other departments whom we interviewed insisted that they only concern themselves with elementary techniques, such as substitution and integration-by-parts. Other departments seem to have no interest in hyperbolic or complicated rational integrands, and we see no reason that these topics should be treated instead of formal mathematics, more applications, or other new topics.
- **Weak Emphasis on Rigor.** According to the department web-page, "one of the goals of Math 32 is for students to develop an understanding and appreciation of the formal structure of calculus." However, formal and technical topics such as the Fundamental Theorem are are not reviewed, emphasized, or tested. Simply lecturing the proof of this theorem does *not* force the students to develop understanding or appreciation. Similarly, convergence of series is a subtle and formal topic which takes

significant patience and detailed exercises. The lecture-only format of 32 does not induce the students to carefully investigate these ideas. Furthermore, when formal topics are encountered in the course, students are only expected to apply resulting rules and formulas on the exam, but they are not asked to prove or explain them.

- **Mundane and Repetitive Examples.** Physical applications only arise in this course in two places. First, the relationship between force and work is used as an example of integration. Second, the growth and decay of populations and Newton's law of cooling are used as examples of separable differential equations.

  In the force and work section, far too much time is spent on problems involving filling and emptying tanks of water. The physical, biological, and social sciences contain a mind-boggling variety of interesting integration problems, yet most of the examples and exercises in this course rely only on barrels, jugs, and hoppers.

  The growth and decay problems are also somewhat repetitive, but this is primarily because no time is spent on broader classes of differential equations. We would like to see a more thorough treatment of differential equations, as appears throughout 31L and 32L.
- **Lack of Scope.** This course covers only integration, basic separable differential equations, series, and power series. For a course intended to complete a student's knowledge of single-variable calculus, this seems inadequate. Many topics are absent which are important both to mathematicians, scientists, and engineers. Most notably absent are Fourier series, complex numbers, and more advanced differential equations.
- **Textbook Selection.** Edwards and Penny is generally unpleasant to read. Students (and teachers) find it dry and un-intuitive. Furthermore, it attempts to cover some topics (such as ln and exp) from multiple perspectives without bothering to tell a consistent story and resolve discrepancies. As noted before, this book also has a limited selection of physical applications and does not deal with some important topics.

To solve all of these problems, we suggest a new course, 41L, that will help students gain a broader and more rigorous understanding of single-variable calculus topics and how they are applied in other disciplines. This course will have less repetition from high school, but will still give students the review and practice they need through applications.

CHAPTER 7

41L: Proposed Calculus for Freshmen with AP Credit

1. Summary

Under the current system, freshmen with AP credit for 31 are given the choice to take either 32 or 32L. Neither course completely serves the needs of the freshmen whose AP background has prepared them to do calculations, but has not given them a deeper understanding of calculus. The current 32 course spends a lot of time on complicated integration techniques, half of which the freshmen mastered in their AP course; the other half are rarely used in later mathematics courses or other disciplines. The 32L course assumes that the freshmen understand the definition of the integral and have seen differential equations from 31L. The current 32L course also contains sophomores with weaker mathematics backgrounds whose needs are much different than the freshmen. Neither course is producing mathematics majors, and the difference in performance for freshmen who continue into 103 is negligible. (See Table 3 in Appendix A and Table 3 in Chapter 8.) We propose that all freshmen with the appropriate AP credit be placed in a new laboratory based calculus course (which we have titled 41L) with the following goals:

- Engage the students early with new advanced theoretical topics such as sequences, series and limits. Then reinforce these concepts with topics like the definition of the integral and Taylor series.
- Increase students' understanding of differentiation and integration from the AP level.
- Give students practice on integration techniques by introducing more applications of integration.
- Introduce differential equations and modeling in a self-contained unit.
- Demonstrate how mathematics is used within other disciplines and increase interest in mathematics itself through labs and introducing topics such as differential equations, modeling and Fourier series.

After an experimental section of 41L is taught in the Fall, we will revisit this report and make a recommendation about whether it should permanently replace 32 and 32L for freshmen with AP credit for 31.

2. Background and Motivation

Currently, freshmen who have scored a 5 on the AB Calculus test or a 3 on the BC Calculus test are given the choice to enroll in either Math 32 or 32L. Students who have taken either 26L or 31L in the spring also enroll in 32L. This system does not serve the needs of the students for the following reasons, which are categorized by three distinct student groups.

- Sophomores in 32L:
  - Sophomores from 26L and spring 31L are expected to perform at the same level as freshmen who scored high on the AP exam, but they mostly fall short. (See Table 2 in Appendix A.)
  - Some topics and labs are repeated from 26L.
- Freshmen in 32L
  - This course contains a section on differential equations that is a continuation of the topics covered in 31L, but depending on their background the freshmen may not have seen all of the topics they are expected to know from 31L.
  - Freshmen don't fully understand the definition of the integral which is covered in 31L and 32, but not 32L.
  - Freshmen repeat topics on integration techniques that they mastered in high school.
  - The pace of 32L can be slow at times for freshmen due to repeated high school topics. This generally leaves them disinterested.
- Freshmen in 32
  - This class does not produce many mathematics majors although is recommended for those interested in mathematics.
  - Freshmen repeat topics on integration techniques that they mastered in high school.
  - Too much time is spent on integration techniques and hyperbolic trig functions which are rarely used in later mathematics courses or in other disciplines.
  - Too much time is spent on problems involving filling tanks. Other applications of integration could be introduced as well.
  - Students are expected to perform calculations for numerical integration and the error from Taylor polynomial approximations without a calculator.
  - This class does not contain the variety of applications seen in 32L.

Our goal is to offer a course that will help students get a deeper understanding of calculus topics and how they are applied in other disciplines. This course will have less repetition from high school, but will still give students the review and practice they need through applications.

3. Textbook

We recommend that Math 41L use "Calculus: Concepts & Contexts: 3" by James Stewart. We note that this is not the multivariate textbook by the same author and hence our selection has no effect on the ongoing search for a new Math 103 textbook.

3.1. Edwards and Penny discussion. Edwards and Penny, a book whose approach to calculus will likely be comfortable to Freshmen, has serious deficiencies in its treatment of series. The seventh edition ties the series topics to integration and differentiation topics strongly, making it difficult for an instructor to use this material in a course that begins with series in week one, rather than week nine. In addition, the differential equations topics, while new, lack phase-planes. Phase-planes figure heavily into the new section on differential equations and it will be more work to create phase-plane supplements for students than it is worth. More generally, the overall coverage of differential equations seems too formal and intimidating for such a potentially intuitive subject. The coverage of integration topics is adequate, but not exceptional.

3.2. Stewart discussion. In general, Stewart is an extremely readable textbook. Ideas are explained concisely but accessibly.

Stewart contains an excellent self-contained chapter on differential equations. It starts out very intuitively by building a few simple models and explaining what the equations mean in words before moving on to a good treatment of logistic growth, systems of equations with phase planes, stable and unstable equilibria, and other topics. Although the book itself does not contain material on 2nd-order equations, there is a supplementary section on the Web (in addition, there is an on-line supplement covering Fourier series).

The section on Riemann sums is also excellent. The proper definition (with arbitrary values picked in each sub-interval) is explained, but it is preceded by a nice intuitive buildup. Error rates are also covered very well.

The chapter on sequences/series is good. Its chief virtue is that it is a separate chapter not tied to other topics, which will match the syllabus as written. In an ideal world, the Ostebee & Zorn treatment would be better. However, we already have quite good supplementary material on series written in the coursepack.

3.3. Other textbooks. We also evaluated Hughes-Hallett, Sallas and Hille, Ostebee and Zorn, and Rogawski. Hugues-Hallett covers all of the topics we wish to include, but not at a challenging enough depth, especially when it comes to Riemann sums. Ostebee and Zorn has an excellent treatment of sequences and series and good, challenging integration topics, but lacks differential equations. Rogawski scatters DE topics about different sections, but this is useless for a course with a focused section on the topic. Sallas and Hille, while having challenging problems, a good theoretical approach to the material, and an excellent treatment of sequences and series, is probably inaccessible to students. The language is intimidating and the formalism perhaps too high for the audience of our course.

4. Syllabus

SEQUENCES AND SERIES [Total 2 1/3 weeks, 2 labs]

1-1 Intro to Infinity
1-2 Sequences and Limits
Lab: Limits of Sequences **NEW**
1-3 Series and Sigma Notation

2-1 Series Convergence and the nth Term Test
2-2 p-series and Alternating Series
Lab: Probability and Geometric Series
2-3 Comparison Test, Absolute Convergence, and the Limit Comparison Test

3-1 Ratio Test
3-2 Review for Test #1 (Series Convergence Worksheet)
Lab: Test #1

INTEGRATION [total 4 1/3 weeks, 4 labs]
3-3 Review of Differentiation

4-1 Optimization and Related Rates
4-2 Riemann Sums (Measuring Distance Traveled?)
Lab: Riemann Sums
4-3 Definition of the Integral

5-1 Properties of the Integral
5-2 Mean Value Theorem
Lab: Approximations of the Integral **NEW**
5-3 Fundamental Theorem of Calculus

6-1 Fundamental Theorem of Calculus Part 2
6-2 Techniques of Integration: Substitution and Parts
Lab: More Techniques of Integration: Even and Odd Functions, Partial Fractions and Trig Functions **NEW**

APPLICATIONS OF INTEGRATION [total: 2 weeks, 1 lab]
6-3 Volume and Arc Length

7-1 *Fall Break*
7-2 Force and Work Problems
Lab: Improper Integrals
7-3 The Integral Test

8-1 Continuous Probability
8-2 Review for Test #2
Lab: Test #2

SERIES AND FUNCTIONS [total: 2 weeks, 2 labs]
8-3 Power Series

9-1 Taylor Series
9-2 Using Taylor Series
Lab: Remainder Estimate for Taylor Series **NEW**
9-3 Fourier Series

10-1 Fourier Series
10-2 Fourier Series
Lab: Fourier Series

DIFFERENTIAL EQUATIONS [total: 4 1/3 weeks, 3 labs]
10-3 Differential Equations and Growth/Decay Problems

11-1 Separation of Variables and Linear Models (Interest)
11-2 More Linear Models: Newton's Law of Cooling, Falling Objects
Lab: Chemical Rate Equations
11-3 Nonlinear Models: Logistic Growth, Escape Velocity

12-1 Nonlinear Models
12-2 Review for Test #3
Lab: Test # 3
12-3 Phase Planes

13-1 Euler's Method
13-2 *Thanksgiving*
Lab: Thanksgiving
13-3 *Thanksgiving*

14-1 Nonlinear Systems: Predator-Prey
14-2 Nonlinear Systems: SIR
Lab: Limited Immunity in Epidemics (SIRS)
14-3 Introduction to Complex Numbers

15-1 Oscillations
15-2 Damped Oscillations
Lab: Oscillations in Physiology
15-3 Review for Final and TCE

5. Advantages of the New Course

Students in 41L will see a more rigorous treatment of limits of sequences, differentiation, and integration than in either of the old courses. Our experience with freshmen in these courses shows that they understand how to perform calculations, but do not fully understand the concepts of derivative and integral and how they are related. The definition of the integral will also be emphasized by applications such as computing volumes, arc length, work and probabilities, all of which are limits of Riemann sums.

This course begins with sequences and series, which is a new topic for freshmen who took AP Calculus (AB) in high school. This will give the students something new and interesting to think about instead of reviewing topics on integration that they think they understand from high school. Students will also be familiar with $\sum$ notation when we introduce the definition of the integral in the next section. The course will spend some time both defining the integral and proving the Fundamental Theorem of Calculus. This is an advantage over 32L, where the syllabus expects that they already understand the definition and theorem from high school.

This course will briefly review integration techniques from AP Calculus, but will not go much further. The freshmen will be expected to pass a Gateway test with 90% proficiency in order to pass the course. The Gateway is not scheduled into the syllabus, and the students will be expected to study the integration techniques on their own time and pass the Gateway in the Help-room. Although less time will be devoted to the integration techniques themselves, the students will still get plenty of integration practice throughout the rest of the course. In addition, the students will see the necessity of each of the integration techniques through the applications in this course. In particular the integrals involved in Fourier series contain trigonometric functions, even and odd functions, and often require integration by parts. The section on differential equations will include the logistic growth model, which requires partial fractions when solved by separation of variables.

The section on differential equations will introduce students to how derivatives are used in modeling in several disciplines. Hopefully this will increase their interest in mathematics and get them to enroll in other math courses in their future. The course will emphasize what it means to be a solution of a differential equation, which is something that students struggle with in later courses both inside and outside the mathematics department. Also exposure to differential equations early will help them to be more successful in the later differential equations courses. This section does not require the students to have any previous background in differential equations. The has an advantage over 31L and 32L where the topics are spread sporadically throughout a year-long sequence.

CHAPTER 8

102 & 103: Multivariable Calculus

1. Summary

The make-up of Calculus III is changing due to demands by the economics department, a changing attitude in the department toward the requirement of multivariable calculus before linear algebra, and the sheer number of incoming freshman taking 103. With the advent of 102, and the retooling of 104 to accept bright freshmen, the population of 103 students is more heavily Engineering, Chemistry, and Physics majors than ever before. As the new courses evolve to funnel students out of the traditional 103 course, the traditional 103 course should also be changed to improve the student experience.

2. Observations and Concerns

While not our primary avenue of investigation, discussions with the Economics, Civil Engineering, and Biomedical Engineering departments and with faculty members Clark Bray, Lewis Blake, and Margaret Hodel gave us insight into the changes planned for Calculus 3. With the advent of 102 and the retooling of 104 to accept bright freshmen, the population of 103 students is more heavily Engineering, Chemistry, and Physics majors than ever before. Although the current course is already heavily focused toward applications in physics, it is still jammed with material and doesn't give students time to digest the culminating subjects of vector calculus.

The new 102 course will remove most economics majors from 103, while the re-working of 104 will remove many math majors and probably computer science majors. Previously, CS majors took 103 not for its content, but to gain access to 104. Other departments should be informed about the math department's plans for 104 to adjust their requirements.

Math 103 now will be more focused on students interested in classical vector calculus topics. In conversations with civil and mechanical engineering, applications in fluid mechanics were important, and students need a good background in the Divergence theorem. Also, students concurrently taking electricity and magnetism (Physics 42L, 54L, 62L) in the Physics department need to understand differentiating and integrating electric fields and using Gauss's theorem. Higher level E&M courses require the equivalent of 107, as does the Physics major. The same applies to electrical engineers, who need 103 as a prerequisite for 107 and for ECE 53L, their E&M course. A significant number upper-level ECE courses also require 135. Chemistry majors need 103 for physical chemistry, which includes topics in quantum mechanics. A summary of the subjects needed by majors in other departments is shown in Table 2.

Graduate students have enjoyed teaching 103 in the past and would like to continue to be given the opportunity to teach 103. We have no objection to seeing linear algebra being taught in parallel with multivariable calculus and would like to help in the construction of

Major	**Application(s)**	**103 Topic(s)**
Economics	Multivariable optimization	Differentiation Multivariable optimization Lagrange Multipliers
Physics	Electric fields Work Pre-requisite for 107	Differentiation and integration Vector fields, Line integrals Green's (all versions)
Electrical Engineering	Pre-requisite for 107	Differentiation and integration Vector fields, Green's (all versions)
Civil Engineering	Pre-requisite for 107	Differentiation and integration Vector fields
Mechanical Engineering	Pre-requisite for 107 Fluid mechanics	Differentiation and integration Vector fields Divergence Theorem
Biomedical Engineering	Pre-requisite for 107	Differentiation and integration Vector fields
Chemistry	Physical Chemistry Quantum Mechanics	Differentiation and integration Vector fields
Computer Science	Pre-requisite for 104	Vector fields

Table 1. A summary of the subjects needed by different majors from Multivariable Calculus (Math 103).

the course. The 103 curriculum as it now stands is very fast-paced, and either additional sessions or a smaller set of subjects should be considered to help students digest the material. Additionally, if 41L is adopted, Edwards and Penny will be less useful as a textbook since it may not be used in two courses. A new book can be chosen for 103, free of the need to be strong in single-variable calculus topics.

CHAPTER 9

Administrative Recommendations

In addition to the specific course recommendations detailed elsewhere, our investigation of the calculus curriculum has led us to several conclusions regarding the general operations of entry-level instruction and the structure of the calculus committee which oversees that instruction.

1. Graduate Instruction

Many of our PhD candidates are keenly interested in the teaching component of an academic career. In addition to keeping us fed, teaching these courses provides a crucial component of our professional development.

To this end, experienced and interested graduate instructors should be given the option of running the lab associated to their lecture. Especially for those interested in a teaching career, this option allows for finer control over their section and an opportunity to experiment with syllabus and lab changes. In addition to building the résumé of the PhD candidate, this would help drive innovation within the courses. Also, a consistent experience would doubtlessly benefit the students in such sections.

For similar reasons of professional development, many PhD candidates are interested in teaching more advanced courses, such as 102, 103, 104, 107, and 108. While these courses are undergoing widespread changes, we ask that they are not re-designed in such a way that makes graduate instruction impossible.

2. Course Oversight

We formed this review committee and investigated the curriculum; because, there was significant annoyance among graduate instructors that many well-known problems had gone un-addressed for years, and the syllabuses of the various courses had drifted or become stagnant over time.

Every course should always have a coordinator (either a faculty member or an experienced and willing graduate instructor), who makes sure that the various lecturers and lab instructors and assistants are on-track and aware of potential pitfalls with upcoming material. Such oversight keeps young graduate students from drowning among their many deadlines and responsibilities. It also helps the course maintain a consistent quality over time, and it spurs innovation in the syllabus.

Overwhelmingly, the calculus courses are taught by graduate students, and in the appendix to this document, the reader will note that many graduate students have accumulated significant experience with these courses over time. Many graduate students know the material of certain courses intimately, and can point to particular exercises and examples which consistently confound their students. We are pleased that there are currently two graduate positions on the calculus committee, and we strongly urge hope that the committee will make these positions permanent so that our experience can be brought to bear when considering the calculus curriculum. Additionally, we ask that experienced graduate instructors be directly involved when homework lists, labs, and textbooks are reviewed each year.

Finally, the calculus committee should keep and track data regarding students' test scores, majors, and enrollment history and correlate these with each-other and with the performance of these students in calculus courses. The AP system has a growing and changing influence on the background of our students, and the ultimate careers of these students have growing and changing mathematical requirements. *If we have any hope of ultimately serving our students, then this demographic data must be provided to and discussed by the department on a regular basis.*

In the immediate future, such data should be used to reassess the AP placement system, as there is some doubt among us that a score of 3 on the BC exam should give credit for 31.

APPENDIX A

Enrollment and Performance Statistics

1. Freshman in 32L

These data indicate performance gaps between freshmen and upperclassmen in 32L. Here, upperclassmen refers to students beyond their first year. *(Source: University Registrar via Jack Bookman)*

During Fall 2004, Fall 2005 and Fall 2006, there were 103 freshmen and 159 upperclassmen who took Math 32L. All D's F's and W's (withdraws) went to upperclassman.

Table 1 contains two sets of numbers. The numbers in the Observed table are the actual numbers; the numbers in the Expected table are the numbers you would expect to see if there were no difference in performance (e.g. since 103/262 of the students were first years they should have gotten 103/262 of the 138 A's and B's — 54.25). The chi-squared figure of 44.50 indicates that the probability is less than 0.1% that these differences between the observed and expected scores would have occurred randomly with two groups of identical students.

Table 2 are the mean grades — 3.05 for first years, 2.39 for upperclassmen. An A corresponds to a score of 4, a B to 3, a C to 2, a D to 1 and an F to 0. The two-sample z-test score of 6.7 indicates that the difference in the scores is highly statistically significant; that is, the probability that this data was randomly selected from a Normal distribution is less than 0.1%.

F stands for freshmen and U stands for upperclassmen. The grades have been grouped into A and B (AB), C, and D, Fail and Withdraw (DFW).

Observed	AB	C	DFW	Total		**Expected**	AB	C	DFW	Total
F	77	26	0	103		F	54.25	32.63	16.12	111.76
U	61	57	41	159		U	83.75	50.37	24.88	150.24
Total	138	83	41	262		Total	138	83	41	262

TABLE 1. Observed (left) and Expected (right) Data

$$\chi^2 = 44.50 \text{ with df}= 2.$$

	μ	σ	n
F	3.05	0.725	103
U	2.39	0.863	159
		z score	6.70

TABLE 2. Grade Data for F and U

2. Performance in 103

These data compare performance in 103 of those students from 32, 32L, and 41. *(Source: University Registrar via Jack Bookman)*

> "As part of an initial investigation about effectiveness of our first year program, I examined data, that Lewis obtained from the Registrar for this purpose, about two groups of students, students who took Math 103 in Spring 05 and either Math 32, 41 or 32L in Fall 04 and students who took Math 103 in Spring 06 and either Math 32, 41 or 32L in Fall 05.
>
> For each of these students, I requested their grades in Math 32, 41 or 32L, Math 103 and scores on the Math SAT, ACT and Advanced Placement Calculus Tests (AB and BC).
>
> I have examined one question (of many possible questions): Is there a difference performance in Math 103 among students who had Math 32, 41 or 32L? In short, I found there was no difference in the performance. The data is summarized below for the combined cohorts." – Jack Bookman

μ is the mean grade in Math 103 of the given group, n is the number of students in the given group and σ is the standard deviation of the scores in the given group from the group's mean. Data for each cohort is available from Jack Bookman or from this committee by request.

	μ	n	σ
32L	2.92	45	0.864
32	2.95	152	0.980
41	2.9	65	0.919

Table 3. Calc III grades by Calc II class, combined 2004-2005 and 2005-2006 cohorts

As was noted by Jack Bookman, the difference between the scores in Table 3 is statistically insignificant. In particular, the z-test score between 32L and 32 is 0.20, and the z-test score for 32L and 41 is 0.05. This means that the probability is high that these data are randomly selected from a Normal distribution. However, when SAT scores are examined, it was found that those students entering into Math 32L had a significantly lower SAT score than those entering into Math 32.

3. Sources of Math Majors

This data compare the rates at which mathematics majors are produced from the various calculus courses. *(Source: University Registrar via Jack Bookman)*

> "Below is a table summarizing the first math course taken by the 110 math majors who graduated in 2004, 2005 and 2006. It is no surprise that the largest number (30 or 27%) start in Math 103x and second most in Math 103 (though that number from 103 is a small percentage of those students taking 103). In fact, when I examined the data of all students who graduated in 2006 and who took Math 103 at some point in their careers here, I computed that about 9% of those students became math majors or minors.

What is most notable is how few students (3 or 2.7%) who take Math 32 go on to be math majors. That's one per year. 41 is more successful at producing math majors and it has a smaller enrollment than 32. 32L had the same number (5) as 41 and, although I don't have the exact numbers, we know that very few first year students take 32L in comparison to 41 and 32. More students starting in 31L go on to become math majors than in 32 and 41 combined which is surprising since 32/41 and 31L have roughly the same enrollment in the fall, with perhaps more students taking 32/41 than 31L."
–Jack Bookman

First Math Class taken	Number of Students Becoming Math Majors	Fraction of Total Math Majors
32	3	0.027
41	5	0.045
25L	2	0.018
31L	10	0.091
32L	5	0.045
103	23	0.209
103X	30	0.273
104	10	0.091
104X	4	0.036
104++	14	0.127

Table 4. Calc III grades by Calc II class, combined 2004-2005 and 2005-2006 cohorts

APPENDIX B

Methods

1. Participants

This is a complete list of the individuals who had a direct hand in writing this report. Their teaching experience at Duke is also listed.

- Rann Bar-On
  1. 31L Lab: Fall 2004
  2. 31L Instructor: Fall 2005
  3. 26L Instructor: Spring 2006
  4. 32 Instructor: Fall 2006
  5. 41 Instructor: Spring 2007
- Paul Bendich
  1. 31L Lab: Fall 2004
  2. 31L Instructor: Fall 2005
  3. 32 Instructor: Summer 2006
  4. 32L Instructor: Fall 2006
- Benjamin Cooke
  1. 25L Lab: Fall 2001
  2. 31L Instructor: Fall 2002, Fall 2004
  3. 107 Instructor: Summer 2005
  4. 103 Instructor: Fall 2005
  5. 103 Instructor: Fall 2006
- Michael Gratton
  1. 32L Lab: Spring 2004
  2. 108 Instructor: Summer 2005
  3. 32L Instructor: Fall 2005
- Timothy Lucas
  1. 31L Lab: Fall 2001
  2. 31L Instructor: Fall 2002
  3. 32L Instructor: Fall 2003
  4. 108 Instructor: Fall 2006
  5. 107 Instructor: Spring 2007
- Michael Nicholas
  1. 25L Instructor: Fall 2003, Fall 2005
  2. 32 Instructor: Fall 2004
  3. 32L Instructor: Spring 2007
  4. 108 Instructor: Summer 2006
- Nicholas Robbins
  1. 31L Lab: Fall 2002
  2. 25L Instructor: Fall 2003, Fall 2005
  3. 31L Instructor: Spring 2005
  4. 103 Instructor: Summer 2006
  5. 32 Instructor: Fall 2006
- Abraham Smith
  1. 32L Lab: Spring 2005
  2. 32L Instructor: Fall 2005, Spring 2006
  3. 32 Instructor: Fall 2006
- Joseph Spivey
  1. 32L Lab: Fall 2003
  2. 32L Instructor: Fall 2004
  3. 31L Instructor: Fall 2005
  4. 41 Instructor: Spring 2006
  5. 32L Instructor and Coordinator: Fall 2006

2. Committee Background and Procedure

The idea of this Graduate Student Calculus Committee originated in informal conversations among graduate instructors and teaching faculty during the Fall 2006 semester. It was becoming increasingly clear that the calculus curriculum at Duke was about to enter a transition, and participants in these conversations felt that it would be advantageous to students, graduate instructors, and faculty (the department as a whole) if experienced graduate students were to have significant input into the upcoming reforms.

In a number of subsequent conversations with other graduate students, the ideas behind the formation of the committee were solidified and eventually took shape in a proposal and request for participation, which was e-mailed to all graduate students in December.

The committee met weekly during the Spring 2007 semester to discuss the current calculus curriculum, to invite teaching faculty and ask their views, to consult with members of other departments as to their students' calculus needs, and to produce a final report on the committee's work by the end of March.

Faculty from the following departments were consulted in face-to-face conversations with individual members of this committee: Civil Engineering, Mechanical Engineering, Economics, Physics, Chemistry, Environmental Sciences, Computer Science, and Mathematics. A number of faculty from other departments were also contacted, but they did not respond to our requests.

In order to gain a complete view of the current curriculum and its history, our entire committee interviewed Lewis Blake, Jack Bookman, Clark Bray, Margaret Hodel, and Jim Tomberg.

After these interviews, we all carefully discussed each course in the curriculum and debated changes large and small. Finally, those who had particular interest in each course wrote proposals, which the entire committee carefully reviewed. These proposals comprise this document. Overall, the entire committee is pleased with the result, and we are confident that it represents a clear improvement in the calculus curriculum.

3. Acknowledgments

The committee wishes to thank all the teaching faculty of the Duke Mathematics Department for meeting with us and helping us understand the curriculum and its history. We also wish to thank those members of other departments who took time to meet with committee representatives and help us understand the role of mathematics in their classes.

Special thanks go to Jack Bookman for gathering statistical data and helping us understand the process and agenda of the Calculus Committee so that this report can be as helpful as possible. Finally, we wish to thank Lewis Blake for his constant advice and tireless advocacy during the formation of this report.